Optimum projection angle for attaining maximum distance in a soccer punt kick

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Abstract
To produce the greatest horizontal distance in a punt kick the ball must be projected at an appropriate angle. Here, we investigated the optimum projection angle that maximises the distance attained in a punt kick by a soccer goalkeeper. Two male players performed many maximum-effort kicks using projection angles of between 10° and 90°. The kicks were recorded by a video camera at 100 Hz and a 2-D biomechanical analysis was conducted to obtain measures of the projection velocity, projection angle, projection height, ball spin rate, and foot velocity at impact. The player’s optimum projection angle was calculated by substituting mathematical equations for the relationships between the projection variables into the equations for the aerodynamic flight of a soccer ball. The calculated optimum projection angles were in agreement with the player’s preferred projection angles (40° and 44°). In projectile sports even a small dependence of projection velocity on projection angle is sufficient to produce a substantial shift in the optimum projection angle away from 45°. In the punt kicks studied here, the optimum projection angle was close to 45° because the projection velocity of the ball remained almost constant across all projection angles. This result is in contrast to throwing and jumping for maximum distance, where the projection velocity the athlete is able to achieve decreases substantially with increasing projection angle and so the optimum projection angle is well below 45°.

Key words: Sports biomechanics, sports projectile.

Introduction
A long punt kick by a soccer goalkeeper is a very useful skill (Figure 1). Most experienced keepers are able to kick the ball from within their penalty area to well beyond the halfway line and so turn a defensive situation into an offensive one. The farther the keeper can kick the ball the larger the area in which his/her team mates may receive the ball and the greater the offensive opportunities. It is well known that a greater projection velocity results in a greater kick distance (de Mestre, 1990; Wesson, 2002). However, the optimum projection angle to achieve maximum distance in a punt kick is less clear.

In previous experimental studies of throwing and jumping the optimum projection angle was not 45° as is sometimes supposed. The optimum projection angle to be 45° the athlete’s projection velocity must be the same at all projection angles. Even a small dependence of projection velocity on projection angle is sufficient to produce a substantial shift in the optimum projection angle (Hubbard, 2000). If the athlete’s projection velocity increases with increasing projection angle then the optimum projection angle will be greater than 45°, and if the athlete’s projection velocity decreases with projection angle then the optimum projection angle will be less than 45°.

The optimum projection angles in studies of throwing and jumping events were 28–34° for the shot put (Hubbard et al., 2001; Linthorne, 2001; Maheras, 1995), 30–35° for the javelin throw (Red and Zogaib, 1977; Viitasalo et al., 2003), 36–39° for the discus throw (Leigh et al., 2010), 26–32° for the soccer throw-in (Linthorne and Everett, 2006), 21–25° for the long jump (Linthorne et al., 2005), and 19–27° for the standing long jump (Wakai and Linthorne, 2005). In these studies the projection velocity that the athlete was able to produce was greater at low projection angles than at high projection angles and so the optimum projection angle for the athlete was well below 45°. Three factors were responsible for the greater projection velocity at low projection angles: (1) when using low projection angles the athlete does not need to overcome as great a fraction of the weight of the projectile (Linthorne, 2001; Wakai and Linthorne, 2005); (2) in throwing, the musculoskeletal structure of the human body is such that the athlete can generate more throwing force in the horizontal direction than in the vertical direction (Linthorne,
2001; Linthorne and Everett, 2006); and (3) the athlete may transfer velocity from a run-up to increase the projection velocity (Linthorne et al., 2005). The flight of a projectile is also affected by the aerodynamic properties of the projectile and by the height difference between launch and landing. However, in the events studied previously the effects of aerodynamics and launch height on the optimum projection angle were relatively small.

All the previous experimental tests of the optimum projection angle were either of throwing events (shot put, javelin throw, discus throw, soccer throw-in) or jumping events (long jump, standing long jump). Kicking is another fundamental class of human movement but optimum projection angles in kicking activities (e.g., punt kicks and instep kicks in the various football codes) have not been experimentally tested. In the study reported here, we investigated the optimum projection angle in a punt kick by a soccer goalkeeper.

To the best of our knowledge there are no published data on projection angles in a soccer punt kick for maximum distance. However, a simple biomechanical argument suggested that the optimum projection angle in a punt kick might be slightly greater than 45°. As the projection angle in a punt kick is increased, we expected that the player’s foot would make contact with the ball at a greater height above the ground because the kicking leg has rotated further upwards about the hip (Hay, 1993). That is, in kicks at high projection angles the player would use a longer angular distance over which to accelerate the foot (assuming that the player starts rotating the kicking leg from the same hip angle). Therefore, we expected that the projection velocity in a punt kick would increase slightly with increasing projection angle and so the optimum projection angle would be greater than 45°. Also, we expected that changes in projection height would have only a small effect on the optimum projection angle because the range of the kick is very much greater than the height difference between launch and landing (Hay, 1993; Linthorne, 2001). In a soccer punt kick the ball is a moderately aerodynamic projectile and so we expected that aerodynamic effects would have a large effect on the range of the kick but only a slight influence on the optimum projection angle (de Mestre, 1990; Linthorne and Everett, 2006).

In the study reported here, we used a high-speed video camera to measure the projection variables (i.e., projection velocity, projection angle, projection height, and ball spin rate) of maximum-effort kicks by two male soccer players. The kicks were performed over a wide range of projection angles. Our hypothesis was that the player’s optimum projection angle could be calculated by substituting mathematical equations for the measured relationship between the player’s projection velocity and projection angle (and the relationships between the other projection variables) into the equations for the aerodynamic flight of a soccer ball. If the relevant flight mechanics and the relevant relationships between the projection variables were accounted for, the calculated optimum projection angle would be in good agreement with the player’s preferred projection angle. We expected the player’s projection velocity to increase slightly with increasing projection angle and that the optimum projection angle would therefore be greater than 45°.

**Methods**

In a punt kick the kick distance (or horizontal range) \( R \) is the horizontal distance the ball’s center of mass travels from the instant of leaving the foot to the instant of landing (Figure 2). The projection variables that determine the kick distance are the projection velocity, \( v \), the projection angle, \( \theta \), the relative projection height (i.e., the height difference between the projection and the landing), \( h \), and the spin rate of the ball, \( \omega \) (which determines the aerodynamic lift on the ball). However, these variables are interrelated. To calculate a player’s optimum projection angle we used mathematical equations for the observed relationships between projection velocity and projection angle, \( v(\theta) \), between relative projection height and projection angle, \( h(\theta) \), and between ball spin rate and projection angle, \( \omega(\theta) \). These equations were then used as inputs in the equations for the aerodynamic flight of a soccer ball. The player’s optimum projection angle was the angle at which the calculated kick distance was the greatest. Intervention was used to obtain measurements of the player’s projection variables over a wide range of projection angles rather than just at the player’s preferred projection angle. This was necessary to produce a sufficiently low
uncertainty in the mathematical relationships between the projection variables and hence allow a reliable calculation of the player’s optimum projection angle.

**Participant and kicking protocol**

Two semi-professional male soccer players volunteered to participate in the study. Participant 1 was 21 years, height 1.73 m, and weight 78 kg; and Participant 2 was 21 years, height 1.80 m, and weight 76 kg. The study was approved by the Human Ethics Committee of Brunel University, the participants were informed of the protocol and procedures prior to their involvement, and written consent to participate was obtained. The kicks were conducted in still air conditions in an outdoor stadium using a FIFA approved match ball (Nike English Premiership, size 5) that was inflated to the regulation pressure. All kicks were performed from a flat grass surface and the landing area was level with the projection surface. The participants wore athletic training clothes and their own football boots. In this study the participants were instructed to use a ‘straight ahead’ run-up and kicking action (i.e., the run-up and action of the kicking leg were in the plane of the flight of the ball). In all trials the participants dropped the ball from about chest height and then kicked the ball without the ball bouncing on the ground.

Participant 1 performed seven maximum-effort kicks at his preferred projection angle and 51 maximum-effort kicks at other projection angles that ranged from ‘much higher’ to ‘much lower’ than his preferred projection angle. The order of the projection angles was altered to preclude any effect resulting from the order, and an unlimited rest interval was given between kicks to minimise the effects of fatigue on kicking performance. For each kick the kick distance was measured to the nearest 0.1 m using a fibreglass tape measure. In this study the participants elected to use a ‘shorter run-up’ and kicking action (i.e., the run-up and action of the kicking leg were in the plane of the flight of the ball). In all trials the participants dropped the ball from about chest height and then kicked the ball without the ball bouncing on the ground.

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**Video analysis**

A JVC GR-DVL 9600 video camera (Victor Company of Japan, Yokahama, Japan) operating at 100 Hz and with a shutter exposure time 1/500 s was used to record the movement of the ball and player during the kicks. The video camera was mounted on a rigid tripod placed at right angles to the kick direction about 22 m away from the plane of the kick. The field of view was zoomed to allow the participant and ball to be in the field of view throughout the run-up and kick and for at least 10 frames after impact. The movement space of the video camera was calibrated with nine calibration points on three vertical calibration poles that were placed along the line of the kicking plane and 2 m apart.

An Ariel Performance Analysis System (Arial Dynamics, Trabuco Canyon, CA, USA) was used to manually digitise the motion of the player’s kicking limb and the center of the ball in the video images. All digitising was performed by the same operator so as to maximise the consistency of the measured values. Markers were placed on the player’s skin or clothing directly over the joint centers of the left shoulder, hip, knee, ankle, and toe. Each trial was digitised from about 2 steps before the kick to about 10–20 frames after ball impact. The two-dimensional coordinates were calculated from the digitised data using the two-dimensional direct linear transform (2D-DLT) algorithm. Coordinate data were smoothed using a second-order Butterworth digital filter with a cut-off frequency of 10 Hz for the horizontal direction and 12 Hz for the vertical direction, and the velocities of the player joint markers were calculated by numerical differentiation of the coordinate data (Winter, 2009). The choice of cut-off frequency was based on a visual inspection of the time traces and power spectra of the raw and filtered coordinate data. The instant of foot-ball impact was defined as the last frame before the ball was observed to come in contact with the player’s foot, and the instant of projection was defined as the first frame in which the ball broke contact with the player’s foot. The foot velocity was represented by the velocity of the foot’s center of mass, which was defined as the midpoint of the ankle and toe markers.

In a soccer kick the filtered velocity data do not produce accurate measures of the ball projection velocity because of the rapid change in the velocity of the ball during the foot-ball impact (Knudson and Bahamonde, 2001). Instead, the ball projection velocity was calculated using unfiltered ball displacement data from images immediately after the instant of projection. The horizontal component of the ball velocity was calculated as the first derivative of a linear regression line fitted to the unfiltered ball displacement data, and the vertical component of the ball velocity was calculated as the first derivative of a quadratic regression line (with the second derivative set equal to $-9.81 \text{ m/s}^2$) fitted to the unfiltered ball displacement data (Nunome et al., 2006). Eight images were used in the calculation of the ball velocity components as this gave the best compromise between the decrease in uncertainty and the decrease in accuracy (due to the increasing effect of aerodynamic drag) as the number of data points was increased. The projection velocity and projection angle of the ball were calculated from the horizontal and vertical velocities of the ball. The projection height was the vertical distance of the center of mass of the ball relative to the ground at the instant of projection, and the projection distance was the horizontal distance of the center of mass of the ball relative to the kicking line at the instant of projection. The effective kick distance was calculated by adding the projection distance to the measured kick distance. The spin rate of the ball was calculated from the rotation of the ball markings (three hoops) over 5–20 frames after impact. As this was a 2-D study, the ball spin was only calculated for a spin direction that was perpendicular to the plane of the kick (i.e., backspin or topspin).

The uncertainties arising from fitting curves to the
unfiltered ball displacement data were 0.2 m·s⁻¹ for projection velocity and 0.2° for projection angle. The standard deviations in the projection variables arising from re-digitising a trial five times were 0.05 m·s⁻¹ for projection velocity, 0.006 m for projection height, and 0.2° for projection angle. In this study the greatest source of uncertainty in the projection height arose from the sampling frequency of the video camera, and this uncertainty was taken as one half the difference between the value at the instant of projection and the value at one frame before the instant of projection (0.07 m). The uncertainty in the ball spin rate was about 0.3 rev/s.

Model of the aerodynamic flight of a soccer ball
We analysed the trajectory of the soccer ball in a rectangular coordinate system where the positive x-axis was in the forward horizontal direction, the positive y-axis was vertically upwards, and positive spin was anticlockwise (Figure 2). The aerodynamic flight trajectory equations of the soccer ball were then (Bray and Kerwin, 2004; de Mestre, 1990)

\[
\frac{dx}{dt} = -kv(C_d \frac{dx}{dt} + C_l \frac{dy}{dt})
\]

and

\[
\frac{dy}{dt} = kv(C_l \frac{dx}{dt} - C_d \frac{dy}{dt}) - g
\]

where \(\frac{d}{dt}\) and \(\frac{d^2}{dt^2}\) are the first (velocity) and second (acceleration) derivatives with respect to time, \(v\) is the velocity of the ball relative to the air, \(C_d\) is the drag coefficient, \(C_l\) is the lift coefficient, and \(g\) is the acceleration due to gravity (9.81 m·s⁻²). The constant \(k\) is given by \(k = \frac{\rho D}{2m}\), where \(\rho\) is the air density (1.225 kg·m⁻³ at sea level and 15°C), \(S\) is the cross-sectional area of the ball (0.038 m²), and \(m\) is the mass of the ball (0.43 kg).

The lift coefficient of a soccer ball increases exponentially with increasing spin rate from \(C_l = 0\) for no spin up to a limiting value of about \(C_l = 0.25\) (Asai et al., 2007; Bray and Kerwin, 2004; Carré et al., 2002). Here, we used an empirical fit to the experimental data (Linthorne and Everett, 2006; Mestre, 1990)

\[
C_l = -0.25e^{-0.5\omega} + 0.25
\]

where \(\omega\) is the spin rate in revolutions per second.

A spinning ball has a slightly greater drag coefficient than a non-spinning ball. For a spinning soccer ball the drag coefficient increases at a rate of about 0.014 per 1 rev·s⁻¹ increase in spin rate (Asai et al., 2007). That is, the drag coefficient is given by

\[
C_d = C_{d0} + 0.014\omega
\]

where \(C_{d0}\) is the drag coefficient at zero spin. At velocities typical of the soccer kick (25–35 m·s⁻¹), a soccer ball that is projected with zero spin has a drag coefficient of about \(C_{d0} = 0.1–0.2\) (Asai et al., 2007; Carré et al., 2002).

If the initial conditions of the ball (i.e., projection velocity, projection angle, projection height, and spin rate) are known, the trajectory of the ball may be computed and the distance of the kick determined. Because the projection velocity, projection angle, projection height, and ball spin rate are inter-related, we used the measured equations for \(v(\theta), h(\theta),\) and \(\omega(\theta)\) to generate the initial conditions for the flight trajectory equations. The flight trajectory equations are non-linear and so must be computed using numerical methods. In this study we used a technical computing software package (Mathematica; Wolfram Research, Champaign, IL, USA) to calculate the flight trajectories. The calculated kick distance was plotted against projection angle (for angles from 0° to 90°), and the optimum projection angle was the point on the curve at which the kick distance was greatest.

Uncertainty in the calculated optimum projection angle
The uncertainty in the calculated optimum release angle was calculated using the bootstrap method (Efron and Tibshirani, 1993). The participant’s kick trials were randomly re-sampled to obtain 10 new data sets. The re-sampling was performed using the standard deviation of the y-residuals of the curves fitted to plots of \(v(\theta), h(\theta),\) and \(\omega(\theta)\) for the original data set. For each of the 10 new data sets the optimum projection angle was re-calculated using the method described previously. The 95% confidence interval in the mean optimum projection angle for the 10 re-sampled data sets was taken as the uncertainty in the calculated optimum projection angle.

Kicking mechanics
In this study, we also analysed the player’s kicking mechanics with the aim of explaining the observed relationship between projection velocity and projection angle, \(v(\theta)\). In a soccer kick, the projection velocity of the ball is mainly determined by the velocity of the player’s foot at impact (Lees and Nolan, 1998). Therefore, we measured the variables that are associated with foot velocity at impact, namely, the rotational range of motion of the thigh, the knee angle at maximum knee flexion, the maximum angular velocity of the thigh, and the angular velocities of the thigh and shank at impact (Ball, 2008). We plotted these kicking variables as a function of the projection angle and compared them to the player’s relationship for the projection velocity, \(v(\theta)\).

Players of varied physical characteristics and skill
In studies of throwing and jumping events, investigators found that the optimum projection angle was not the same for all athletes (Leigh et al., 2010; Linthorne, 2001; Linthorne et al., 2005; Wakai and Linthorne, 2005). For example, for the five athletes in Linthorne’s study of the shot put the athletes had optimum projection angles that differed by 6° because of inter-athlete differences in the shape of their velocity-angle curves. In the present study, we investigated the effects of inter-athlete differences on the optimum projection angle by performing calculations with models of the projection variables and the kicking mechanics variables. The effects of the player’s physical characteristics and kicking skill were calculated by adjusting the values in the models for the player’s muscular strength, body size, run-up velocity, and the spin imparted to the ball.
Results

The mean values for the seven kicks at the preferred projection angle by Participant 1 were: distance, 39.5 ± 2.7 m; projection velocity, 27.1 ± 0.6 m·s⁻¹; projection height, 0.55 ± 0.07 m; projection angle, 43.9 ± 4.0°; and ball topspin rate, 3.9 ± 0.9 rev·s⁻¹ (mean ± s). For the six kicks at the preferred projection angle by Participant 2 the values were: projection velocity, 24.2 ± 0.5 m·s⁻¹; projection height, 0.42 ± 0.05 m; projection angle, 40.1 ± 2.5°; and ball topspin rate, 2.3 ± 0.7 rev·s⁻¹. (We did not measure the kick distances for Participant 2.)

Projection velocity

Contrary to our initial expectations, the kicks by the two participants did not show an increase in projection velocity with increasing projection angle (Figure 3). The projection velocity data suggested a slight inverted u-shape, 
\[ v(\theta) = v_{max} - A(\theta - \theta_{max})^2 \]  
where \( v_{max} \) and \( \theta_{max} \) are the coordinates of the peak of the inverted u-shape, and \( A \) is a measure of the curvature.

A calculation of Akaike’s Information Criterion indicated that a u-shaped curve (i.e., a second-order polynomial) was a substantially better fit to the data than a linear fit (Motulsky and Christopoulos, 2004). A curve of the form of equation (5) was fitted to the plot of projection velocity as a function of projection angle (Figure 3) by selecting values of \( v_{max}, \theta_{max}, \) and \( A \) using the Levenberg-Marquardt algorithm (Press et al., 1988).

Figure 3. Projection velocity as a function of projection angle for a male soccer player. The fitted curve is from equation (5). Data for Participant 1, \( r = 0.51 \). The projection angle remained almost constant across all projection angles and so had little influence on the player’s optimum projection angle.

Table 1. Fitted parameter values of the projection variables and kicking mechanics variables for the two participants. Data are means (± standard error).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fit parameter symbol</th>
<th>Participant 1 (58 kicks)</th>
<th>Participant 2 (25 kicks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projection velocity</td>
<td>( v_{max} ) m·s⁻¹</td>
<td>26.9 (.2)</td>
<td>24.2 (.2)</td>
</tr>
<tr>
<td></td>
<td>( \theta_{max} ) deg</td>
<td>50 (2)</td>
<td>41 (2)</td>
</tr>
<tr>
<td></td>
<td>( A ) (m·s⁻¹)/deg⁻²</td>
<td>.0017 (.0005)</td>
<td>.0021 (.0007)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.51</td>
<td>.57</td>
</tr>
<tr>
<td>Projection height</td>
<td>( h_{leg} ) m</td>
<td>.62 (.03)</td>
<td>.57 (.05)</td>
</tr>
<tr>
<td></td>
<td>( h_{min} ) m</td>
<td>.37 (.01)</td>
<td>.28 (.02)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.94</td>
<td>.92</td>
</tr>
<tr>
<td>Ball spin rate</td>
<td>( \omega_0 ) deg·s⁻¹</td>
<td>-4.5 (.4)</td>
<td>-4.7 (5.7)</td>
</tr>
<tr>
<td></td>
<td>( B_1 ) rev·s⁻¹</td>
<td>.04 (.06)</td>
<td>.11 (4.9)</td>
</tr>
<tr>
<td></td>
<td>( B_2 ) /deg</td>
<td>.06 (.02)</td>
<td>.02 (.04)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.79</td>
<td>.61</td>
</tr>
<tr>
<td>Foot velocity</td>
<td>( v_{max} ) m·s⁻¹</td>
<td>18.0 (.1)</td>
<td>16.4 (.2)</td>
</tr>
<tr>
<td></td>
<td>( \theta_{max} ) deg</td>
<td>44 (5)</td>
<td>46 (3)</td>
</tr>
<tr>
<td></td>
<td>( A ) (m·s⁻²)/deg⁻²</td>
<td>.0007 (.0003)</td>
<td>.0013 (.0005)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.48</td>
<td>.49</td>
</tr>
<tr>
<td>Horizontal hip velocity</td>
<td>( m ) (m·s⁻¹)/deg</td>
<td>-.022 (.004)</td>
<td>.002 (.012)</td>
</tr>
<tr>
<td></td>
<td>( c ) m·s⁻¹</td>
<td>2.6 (2)</td>
<td>2.0 (5)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.56</td>
<td>.04</td>
</tr>
<tr>
<td>Thigh rotational ROM</td>
<td>( m ) deg/deg</td>
<td>.34 (.05)</td>
<td>.01 (.07)</td>
</tr>
<tr>
<td></td>
<td>( c ) deg</td>
<td>89 (2)</td>
<td>78 (3)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.71</td>
<td>.04</td>
</tr>
<tr>
<td>Shank angular velocity</td>
<td>( m ) (deg·s⁻¹)/deg</td>
<td>1.4 (.6)</td>
<td>4.7 (1.2)</td>
</tr>
<tr>
<td></td>
<td>( c ) deg·s⁻¹</td>
<td>1760 (30)</td>
<td>1400 (50)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.30</td>
<td>.04</td>
</tr>
<tr>
<td>Thigh angular velocity</td>
<td>( m ) (deg·s⁻¹)/deg</td>
<td>-.4 (.8)</td>
<td>-1.1 (1.0)</td>
</tr>
<tr>
<td></td>
<td>( c ) deg·s⁻¹</td>
<td>80 (50)</td>
<td>150 (40)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.06</td>
<td>.23</td>
</tr>
<tr>
<td>Knee angle</td>
<td>( m ) deg/deg</td>
<td>.13 (.03)</td>
<td>-.01 (.06)</td>
</tr>
<tr>
<td></td>
<td>( c ) deg</td>
<td>50 (2)</td>
<td>68 (3)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.47</td>
<td>.03</td>
</tr>
<tr>
<td>Max thigh angular velocity</td>
<td>( m ) (deg·s⁻¹)/deg</td>
<td>-.1 (.9)</td>
<td>-.8 (1.4)</td>
</tr>
<tr>
<td></td>
<td>( c ) deg·s⁻¹</td>
<td>1160 (50)</td>
<td>930 (60)</td>
</tr>
<tr>
<td></td>
<td>( r )</td>
<td>.02</td>
<td>.12</td>
</tr>
</tbody>
</table>

\( m \) and \( c \) are the gradient and \( y \)-intercept of a linear fit to the data, and \( r \) is the correlation coefficient.
The calculated values and standard errors of the fitted parameters for the two participants are presented in Table 1, and the fitted curve for Participant 1 is shown in Figure 4. The calculated leg length was shorter than the player’s measured leg length (about 0.95 m) because the kicking leg was not completely straight at the instant of ball impact.

**Ball spin rate**

When kicking a soccer ball it was expected that the player would impart spin to the ball and that this spin would vary with projection angle. Figure 5 shows the effect of projection angle on the ball spin rate for Participant 1. Backspin is denoted by a positive spin rate and produces a positive lift coefficient, whereas topspin is denoted by a negative spin rate and produces a negative lift coefficient. For almost all kicks the participant imparted topspin to the ball. We assumed a relationship of the form

\[ \omega(\theta) = \omega_o + B_1 e^{B_2 \theta} \]  

(9)

where \( \omega_o \) is the ball spin rate for a horizontal projection angle (\( \theta = 0^\circ \)), and \( B_1 \) and \( B_2 \) are constants. This equation gave a good fit to the data with a uniform distribution of the residuals and only three fitted parameters.

A curve of the form of equation (9) was fitted to the plot of ball spin rate as a function of projection angle (Figure 5) by selecting values of \( \omega_o, B_1, \) and \( B_2 \) using the Levenberg-Marquardt algorithm (Press et al., 1988). The calculated values and standard errors of the fitted parameters for the two participants are presented in Table 1, and the fitted curve for Participant 1 is shown in Figure 5.

**Drag coefficient of the soccer ball**

In the calculation of the participant’s optimum projection angle, the flight trajectory calculations required an estimate of the drag coefficient of the ball at zero spin, \( C_{D0} \) (equation 4). In this study we used the measured kick distances for Participant 1 to determine \( C_{D0} \) (Linthorne and Everett, 2006). For each of the kicks by the participant, the range of the kick was calculated by using the measured projection velocity, projection angle, projection height, and ball spin rate as the initial conditions in the calculation of trajectory.
flight trajectory model (equations 1 and 2). The drag coefficient at zero spin in the flight trajectory model was adjusted from 0.10 to 0.20 in increments of 0.01, and the calculated kick distances for each of the kicks by the participant were recorded. Best agreement between the calculated kick distances and the measured kick distances was achieved with $C_D = 0.18$, and this value was taken as the drag coefficient at zero spin in the calculations of the optimum projection angle.

Optimum projection angle
The optimum projection angle for both of the participants was calculated and compared with his preferred projection angle. To calculate the optimum projection angle the values of $v_{\text{max}}$, $\theta_{\text{max}}$, and $A$ were substituted into equation (5), the values of $l_{\text{leg}}$ and $h_{\text{leg}}$ were substituted into equation (8), and the values of $\omega_1$, $B_1$, and $B_2$ were substituted into equation (9). The resulting equations for $v(\theta)$, $h(\theta)$, and $\omega(\theta)$ were then used to generate the initial conditions for the flight trajectory equations (equations 1 and 2) for a series of projection angles between 0° and 90° in steps of 0.01°. For each projection angle the flight trajectory was calculated and the kick distance was recorded. The calculated kick distance was plotted against projection angle, and the optimum projection angle was the point on the curve at which the kick distance was greatest (Figure 6).

Figure 6. Measured kick distance as a function of projection angle for a male soccer player, and the curve of kick distance calculated from the relationships between the projection variables. Data for Participant 1, $r = 0.89$. The curve shows that the optimum projection angle for this player is about 52°.

The calculated optimum projection angles were 52.3 ± 1.1° for Participant 1 and 49.0 ± 1.7° for Participant 2. Although there initially appears to be a discrepancy between these values and the player’s preferred projection angles (43.9 ± 4.0° and 40.1 ± 2.5°, respectively), we note that the kick distance curve has a broad maximum and so is ‘flat’ near the optimum projection angle (Figure 6). It is not necessary for the player to kick at close to the optimum projection angle. Figure 6 shows that Participant 1 will produce a kick distance that is within 5% of the maximum kick distance if he uses a projection angle anywhere between 42° and 62°. That is, Participant 1 will produce a near-maximum kick distance if he uses a projection angle of 52 ± 10°. Likewise, Participant 2 will produce a near-maximum kick distance if he uses a projection angle of 49 ± 9°. We therefore concluded that the calculated optimum projection angles for the two participants were in agreement with their preferred projection angles.

In the bootstrap calculation of the uncertainty in the optimum projection angle for Participant 1, the standard deviations of the y-residuals of the fitted curves to plots of $v(\theta)$, $h(\theta)$, and $\omega(\theta)$ were 0.9 m·s$^{-1}$, 0.06 m, and 1.0 rev·s$^{-1}$, respectively. Similar values were observed for Participant 2. For both participants the uncertainty in the optimum projection angle was mainly determined by the uncertainty in $\omega(\theta)$.

This study confirmed that a player’s optimum projection angle is determined by the relationships between the projection variables [i.e., $v(\theta)$, $h(\theta)$, and $\omega(\theta)$] and by aerodynamic drag and lift. For a non-aerodynamic projectile that is projected at constant velocity from ground level the optimum projection angle is 45°. However, for soccer punt kicks by the participants in this study, the projection velocity that the players could generate remained almost constant across all projection angles. Also, the ball was projected from about 0.6 m above the ground and this projection height increased with increasing projection angle because of changes in the player’s body position at the instant of projection. However, calculations with our models for Participant 1 showed that the player’s velocity-angle relationship, $v(\theta)$, and height-angle relationship, $h(\theta)$, had only a very small effect on the player’s optimum projection angle; they reduced the optimum projection angle by only 1°. A soccer ball experiences substantial aerodynamic drag during its flight through the air. For Participant 1, we calculated that aerodynamic drag reduced the maximum kick distance by 23 m and reduced the optimum projection angle by 3° (compared to a kick in a vacuum). Negative lift arising from the topspin on the ball reduced the maximum kick distance by a further 9 m and increased the optimum projection angle by 11° (compared to a kick with no spin).

An alternative method of identifying the optimum projection angle for a player is to fit a regression curve directly to the measured kick distance versus projection angle data (Figure 6). A third-degree polynomial was found to be the most appropriate polynomial fit to the data for Participant 1, as indicated by a calculation of Akaike’s Information Criterion (Motulsky and Christopoulos, 2004). The calculated optimum projection angle obtained using this polynomial fit (53.9 ± 1.6°) was close to the value (52.3 ± 1.1°) obtained using equations (1), (2), (5), (8), and (9). However, this direct method of determining the optimum projection angle does not shed light on the factors that determine the player’s optimum projection angle.

Sensitivity analysis
The calculated optimum projection angles for the participants in this study were insensitive to the form of the mathematical equations used to express $v(\theta)$, $h(\theta)$, and $\omega(\theta)$, and were insensitive to the value used for the drag
coefficient of the ball (equation 4). For Participant 1 the optimum projection angle obtained with a linear fit to the plot of \(\frac{v}{\theta}\) was within 0.5° of the nominal value obtained using equations (5), (8), and (9). Likewise, the optimum projection angles obtained with second-order polynomial fits to the plots of \(h(\theta)\) and \(\omega(\theta)\) were within 0.1° and 0.5°, respectively, of the nominal value. A drag coefficient at zero spin of \(C_{D0} = 0.15\) increased the optimum projection angle by 0.8°, and a drag coefficient of \(C_{D0} = 0.20\) decreased the optimum projection angle by 0.5°. Reducing the drag coefficient rate to 0.010 per rev·s\(^{-1}\), (equation 4) increased the optimum projection angle by 0.1°.

The calculated optimum projection angle was more sensitive to the assumed mathematical form of the lift coefficient (equation 3). Reducing the lift coefficient limiting value to \(C_{L} = 0.20\) (equation 3) decreased the optimum projection angle by 2.1°, and reducing the exponential rate parameter to –0.3 per rev·s\(^{-1}\) (equation 3) decreased the optimum projection angle by 1.8°.

**Kicking mechanics**

A soccer punt kick is a ‘throw-like’ pattern, where the movement of the proximal (thigh) segment is initiated through muscular torque at the hip joint, with the distal (shank-foot) segment initially lagging behind (Kreighbaum and Barthels, 1996). Later in the kick, momentum is rapidly transferred to the Shank-foot segment, with the thigh segment decelerating to about zero at the instant of ball contact (Figure 1). This ‘whip-like’ action results in the end point of the kinetic chain (i.e., the foot) reaching a very high velocity at the instant of ball contact. When kicking a soccer ball the projection velocity of the ball was expected to be determined by the velocity of the player’s foot at impact and by the coefficient of restitution of the ball-foot collision (Lees and Nolan, 1998). Figure 7 and Table 1 show that the velocity of the player’s foot at impact paralleled the ball velocity. Participants 1 and 2 had a constant ball-foot velocity ratio of 1.48 ± 0.05 and 1.48 ± 0.06, respectively. The constant ball-foot velocity ratio indicates that the coefficient of restitution of the ball-foot collision did not vary with projection angle. The foot velocities in the present study are similar to those reported in a study of punt kicking in Australian Rules football (Ball, 2008). The ball-foot ratios reported here for a soccer punt kick are slightly higher than those reported for a soccer instep kick, probably because in a punt kick
the ball makes contact with the foot closer to the ankle joint and so less energy is lost during the collision due to inelastic deformation of the foot segment (Nunome et al., 2006).

At high projection angles Participant 1 tended to use a shorter and slower run-up and so the horizontal velocity of the player’s hip at ball impact tended to decrease with increasing projection angle (Figure 7). Participant 2 showed no dependence of horizontal hip velocity on projection angle (Table 1). In javelin throwing and cricket bowling the athlete’s run-up velocity contributes to the projection velocity of the implement (Bartlett et al., 1996; Salter et al., 2007). A similar mechanism may apply in a soccer punt kick, but the contribution of the run-up to projection velocity in a punt kick is probably slight because the run-up velocity (0–3 m·s−1) is relatively low compared to the projection velocity of the ball (22–28 m·s−1).

The thigh rotational range of motion of Participant 1 increased slightly with increasing projection angle (Figure 8a). However, this greater rotational range of motion was not accompanied by a greater foot velocity at impact (Figure 7). Participant 2 showed no dependence of thigh rotational range of motion on projection angle (Table 1).

The two participants in this study retained the characteristic whip action of the kicking leg across all projection angles. The player’s thigh angular velocity at impact was always close to zero, indicating a consistent strong transfer of momentum from the thigh to the shank (Figure 8b). The player’s shank angular velocity at impact was almost constant across all projection angles (Figure 8b), in parallel to the trend for the foot velocity at impact. The maximum knee flexion angle during the kick was also almost constant across all projection angles (Figure 8c), as was the maximum thigh angular velocity during the kick (Table 1). The maximum knee flexion angle, maximum thigh angular velocity, shank angular velocity at impact, and thigh angular velocity at impact in the present study were similar to those reported in a study of punt kicking in Australian Rules football (Ball, 2008).

Players of varied physical characteristics and skill
In this study, the kicking mechanics of the two participants were similar, as were the relationships between the projection variables (Table 1), and the calculated optimum projection angles. These similarities suggest that the observed relationships for the two participants were not idiosyncratic and that we should expect relationships of similar form for many other players. To help generalize the findings from the two participants in this study to other players, we used our models of the projection variables and kicking mechanics variables to calculate the optimum projection angle for players with different values for the player’s muscular strength, body size, run-up velocity, and the spin imparted to the ball. In these calculations we used the fitted parameter values obtained for Participant 1 (Table 1) as the nominal values.

A player with a greater muscular strength (particularly in the hip and high muscles of the kicking leg) would be expected to produce a greater maximum foot velocity and hence produce a greater maximum projection velocity (Manolopolous et al., 2006). We investigated the effects of the player’s muscular strength by varying the value of the maximum projection velocity, v_{max}, in equation (5). Our calculations showed that stronger players should be able to kick the ball substantially farther, but they must use a slightly higher projection angle to gain the full benefit from a greater projection velocity (Figure 9a). Note that the range achieved in a punt kick is almost linearly proportional to the projection velocity (rather than proportional to the square of the projection velocity) because of the increasing effect of aerodynamic drag at higher velocities (Wesson, 2002).

Figure 9. Calculated effects of changes in player characteristics and skill on the optimum projection angle and maximum range for a male soccer player: (a) maximum projection velocity; (b) maximum run-up velocity; and (c) maximum topspin rate. These calculations suggested that the optimum projection angle should be around 45–55° for most players.
A taller player would be expected to have a higher projection height because of his longer leg length. We investigated the effect of the player’s body size by varying the value of the player’s leg length, $l_{leg}$ in equation (8). We assumed that the length of an adult male player’s leg is about 53% of his standing height (Winter, 2009). Our calculations showed that differences in the projection height between very short players (1.0 m tall) and very tall players (2.5 m tall) should have a very small influence on the maximum range (<0.2 m) and optimum projection angle (<0.1°).

In the present study, we found that the velocity of the foot at ball impact for Participant 1 depended on his horizontal hip velocity and hence on his run-up velocity (Figure 7). We investigated the effect of the player’s ability to kick the ball when using a high run-up velocity. We assumed that the player’s run-up velocity decreased linearly from a maximum value when using a horizontal projection angle ($\theta = 0°$) down to zero when using a vertical projection angle ($\theta = 90°$). Our calculations showed that players who are able to use a faster maximum run-up velocity should be able to kick the ball slightly farther, but must use a marginally lower projection angle (Figure 9b).

We also examined the effects of ball spin. In the present study, we found that the player almost always imparted topspin to the ball and that the rate of topspin increased at lower projection angles (Figure 5). Topspin reduces the range of a kick and so we supposed that a more technically skilled player would be able to impart less topspin when kicking the ball (possibly through subtle differences in foot position or impact point of the ball on the foot). We assumed that the magnitude of the ball spin rate for a given player decreased exponentially from a maximum value ($\omega_o$) when using a horizontal projection angle ($\theta = 0°$) down to zero when using a vertical projection angle ($\theta = 90°$). That is, we assumed a relationship of the form

$$\omega(\theta) = \omega_o \left[ 1 - e^{-B_1(\theta-90)} \right]$$

where $B_1$ is a constant. (Values of $\omega_o = -5.4 \pm 0.7$ revs$^{-1}$ and $B_1 = 0.21 \pm 0.10$ per degree gave a good fit to the data for Participant 1.)

We investigated the player’s skill in minimising topspin on the ball by varying the value of the maximum spin rate, $\omega_o$, in equation (10). Our calculations showed that players who are able to kick with less topspin should be able to kick the ball farther, but must use a lower projection angle (Figure 9c).

In these calculations we used a wide range of parameter values so as to encompass a large fraction of the population of soccer players. In summary, the results of our calculations suggest that the optimum projection angle in a soccer punt kick should be about 45–55° for most players. The strongest influences on the optimum projection angle are likely to be the player’s strength (i.e., foot velocity) and the amount of topspin imparted to the ball.

**Discussion**

The preferred projection angles for attaining maximum distance in a soccer punt kick by the participants in this study were about 40° and 44°. The optimum angle was close to 45° because the projection velocity that the player could achieve remained almost constant across all projection angles. Participants 1 and 2 had similar kicking mechanics and similar relationships between the projection variables. This, together with calculations made using the models of the projection variables and kicking mechanics, suggested that the optimum projection angle should be around 45–55° for most players.

To the best of our knowledge there are no published data on projection angles in a soccer punt kick for maximum distance. Gómez Piriz et al. (2010) reported an average projection angle of 38.5° for punt kicks by 9 goalkeepers from two Spanish first-division clubs, but the kicks were for accuracy rather than for maximum distance. The kicking action used in the soccer punt kick is similar to that used in other football codes (rugby union, rugby league, American football, Australian football, Gaelic football). The optimum projection angles for the players in the present study were similar to projection angles reported for studies of rugby union and American football. Holmes et al. (2006) reported a projection angle of 43.9 ± 1.5° (mean ± s) for punt kicks by 14 elite rugby union players (with a projection velocity of 28.1 ± 3.7 m·s$^{-1}$ and a kick distance of 55.4 ± 7.2 m), and Smith (1949) reported a projection angle of 47.5° for a punt kick by a collegiate American football player (with a projection velocity of 28 m·s$^{-1}$ and a kick distance of 56 m). Although the balls used in rugby union and American football are not spherical and are a different size to a soccer ball, we expect the effects of aerodynamic drag on the optimum projection angle in these football codes to be similar to that in soccer (i.e., less than a few degrees).

An important practical result from the present study is that projecting the ball at the optimum projection angle is not very important in producing a long punt kick. Kick distance is not sensitive to projection angle and so relatively large errors in projection angle can be tolerated (Figure 6). For the two players in the present study the projection angle only needed to be within about 10° of the optimum projection angle for the kick distance to be within 5% of the maximum achievable distance.

In a soccer punt kick it is much more important for a player to attain a high projection velocity than to kick at the optimum projection angle. The range of a moderately aerodynamic projectile is approximately proportional to the projection velocity (Wesson, 2002). The implication is that to achieve longer kicks the player should work on developing muscular strength to increase the projection velocity (Cabri et al., 1988). An improvement in strength is expected to shift the velocity-angle relationship (Figure 3) upwards, hence producing a greater maximum kick distance at a similar optimum projection angle.

The two participants in the present study preferred to project the ball at about 9° lower than their calculated optimum projection angle. We suspect that using a lower than optimum projection angle is a general characteristic of skilled soccer goalkeepers. A projection angle that is
5–15° lower than the optimum projection angle produces a kick distance that is almost as great as the maximum possible distance, but the flight time of the ball is substantially less and therefore the chance of the kick being intercepted by opposition players is reduced. To illustrate this point, we calculated that a kick by Participant 1 at a projection angle of 42° (10° lower than his optimum projection angle) reduces the flight time by 20% (0.6 s), but the ball still travels 95% of the maximum possible distance.

**Constant projection velocity**

A key finding from the present study is that in a soccer punt kick the projection velocity of the ball remained almost constant across all projection angles. This is in contrast to studies of throwing and jumping events where the projection velocity decreased substantially at high projection angles (Hubbard et al., 2001; Linthorne, 2001; 2005; Linthorne and Everett, 2006; Maheras, 1995; Red and Zogaib, 1977; Viitasalo et al., 2003; Wakai and Linthorne, 2005). Kicking differs from throwing and jumping in that projection velocity is generated during an impact between the athlete and the projectile (i.e., the ball). The projection velocity of the ball is developed over a very short distance (~25 cm) in a very short time (~15 ms), and hence the force exerted by the athlete on the ball is very large (~1 kN) in comparison to the weight of the projectile (Tsoulosidis and Zatsiorsky, 1996). The three factors that cause the projection velocity in throwing and jumping to decrease with increasing projection angle are the weight of the projectile, the musculoskeletal structure of the body, and the run-up. However, in a soccer punt kick these factors are not important because: (1) the weight of the ball is negligible in comparison to the impact force; (2) the foot velocity (and hence impact force) is almost the same across all projection angles, and; (3) although a run-up is used in a soccer punt kick, the horizontal velocity of the player’s hip at impact is relatively small and so the run-up does not make a substantial contribution to the projection velocity of the ball.

**Conclusion**

This study showed that the optimum projection angle in a long soccer punt kick is about 45°. In a punt kick the player uses consistent kicking mechanics across all projection angles and so the player’s foot velocity at ball impact remains constant across all projection angles. This result is in contrast to throwing and jumping for maximum distance, where the projection velocity the athlete is able to achieve decreases substantially with increasing projection angle and so the optimum projection angle is well below 45°. In a soccer punt kick it is not essential for a player to kick the ball at precisely the optimum projection angle because deviations of several degrees do not substantially reduce the distance of the kick. Aerodynamic drag has little effect on the optimum projection angle, but kicking the ball with spin can substantially alter the optimum projection angle as well as the kick distance.

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**References**


Key points

- The optimum projection angle that maximizes the distance of a punt kick by a soccer goalkeeper is about 45°.
- The optimum projection angle is close to 45° because the projection velocity of the ball is almost the same at all projection angles.
- This result is in contrast to throwing and jumping for maximum distance, where the optimum projection angle is well below 45° because the projection velocity the athlete is able to achieve decreases substantially with increasing projection angle.

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