A study of shuttlecock's trajectory in badminton

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Abstract
The main purpose of this study was to construct and validate a motion equation for the flight of the badminton and to find the relationship between the air resistance force and a shuttlecock's speed. This research method was based on motion laws of aerodynamics. It applied aerodynamic theories to construct motion equation of a shuttlecock's flying trajectory under the effects of gravitational force and air resistance force. The result showed that the motion equation of a shuttlecock's flight trajectory could be constructed by determining the terminal velocity. The predicted shuttlecock trajectory fitted the measured data fairly well. The results also revealed that the drag force was proportional to the square of a shuttlecock velocity. Furthermore, the angle and strength of a stroke could influence trajectory. Finally, this study suggested that we could use a scientific approach to measure a shuttlecock's velocity objectively when testing the quality of shuttlecocks. And could be used to replace the traditional subjective method of the Badminton World Federation based on players' striking shuttlecocks, as well as applying research findings to improve professional knowledge of badminton player training.

Key words: Projectile motion, aerodynamics, badminton.

Introduction

Most studies in the past explored the speed of shuttlecocks. Virtually no research has been done on the flight trajectory of the badminton shuttlecock. By understanding the flight trajectory of the badminton shuttlecock we can predict the speed, time, direction and path, and this would provide helpful information for training of players.

Some researchers, for example, Poole (1970), Adrian and Enberg (1971), Gowitzke and Waddell (1979), Tang (1995), Tsai, Huang, and Jih (1995, 1997), Chang (2002), and Hsiao (2005), explored the speed of a shuttlecock. Tsai, Huang, and Jih (1995) analyzed the speed of four different badminton overhead strokes for world class players, and the results of their study showed that the order of shuttlecock speed with respect to stroke was jump-smash, smash, clear, and drop. Obviously stroke force could affect speed of shuttlecocks, and therefore four different badminton overhead strokes resulted in dissimilar speeds. Stroke angle also affected flying direction of shuttlecocks, and consequently four different badminton overhead strokes resulted in varied flying directions. In short, stroke force and stroke angle can affect a shuttlecock's trajectory. Tsai, Huang, and Jih (1997) also conducted research on elite Taiwanese badminton players and suggested that the initial shuttlecock velocities of smashes were 55-70 (m·s⁻¹), with an average of 62.12 (m·s⁻¹); the speeds of jump smashes were 55-75 (m·s⁻¹)

with an average of 68.16 (m·s⁻¹); the clears were 42-51 (m·s⁻¹) with an average of 47.76 (m·s⁻¹), whereas the drops were 22-29 (m·s⁻¹) with an average of 25 (m·s⁻¹). The clear and drop were considerably slower than the smash and jump-smash strokes. Chang (2002) mentioned that while executing stroke actions including smash and jump smash, senior high school badminton players showed a wider extension of the upper arm, a sharper angle at the elbow joint, and an accelerated wrist angular velocity. Moreover, distinct flexing of the wrist is clearly visible. Therefore, the combined actions caused the initial speed in a shuttle to accelerate. Hsiao's research (2005) analyzed the drop position and movement route of badminton players' return shot of a deep clear. He discovered their game strategies by observing badminton games of world's top ranking women's singles players.

Essentially, it is easy to understand that the force and angle of stroke can affect the speed and trajectory of the shuttlecock. Moreover, air resistance force is an important factor that influences performance. It is generally recognized that the Reynolds number $\Re$ play a crucial role in determining whether the law of linear or quadratic air resistance force is used (The Engineering Tool Box, 2005). Moreover, an experiment on vertical fall shows that the best model is the quadratic air resistance force in the instantaneous speed of the falling shuttlecock (Peastrel, Lynch and Angelo, 1980). Therefore, a motion equation of the badminton shuttlecock's flying trajectory could be constructed by determining the terminal velocity in aerodynamics. It is meaningful that we try to construct an aerodynamics equation of the badminton shuttlecock trajectory.

The main purpose of this study is to construct and confirm the motion equation of a shuttlecock's flying trajectory. Applying the equation to badminton, the speed, time, direction and path of a shuttlecock will be able to be predicted, thus contributing to sports training in badminton.

Methods

When a shuttlecock is in flight, according to Newton's Second Law:

$$\vec{W} + \vec{F}_V + \vec{B} = m\vec{a} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (1)$$

In equation (1): $\vec{W}$ is gravitational force, $\vec{F}_V$ is aerodynamic drag force, and $\vec{B}$ is buoyancy. As a matter of fact, the measured air buoyancy of standard shuttlecock, for example which has a volume of 19 cm³ in the air (density~1.205kg·m³), to be about 0.02 gw. We may neglect its

very slight influence, when compared with the force of gravity and aerodynamic drag. The magnitude of aerodynamic drag force depends on the relative speed of the shuttlecock through the air, and its direction is always opposite to the direction of the shuttlecock. Generally, the magnitude of resistance force can be expressed as:

$$F_V = bv^n \text{ (Thornton and Marion, 2003)} \quad \dots \dots \dots \dots (2)$$

Where v is the speed of the shuttlecock relative to air, parameter n is a real number and b is a constant that depends on the properties of the air and the shape and dimension of the shuttlecock. The two parameters b and n in general were determined by experiments.

As a shuttlecock is falling down vertically, the speed and resistance force will increase (Eq. 2). The rate of acceleration becomes zero when the increased resistance force eventually balances the weight. At this point, the shuttlecock reaches its terminal velocity v_T and from then on it continues to move with zero acceleration. After this point, the motion of a shuttlecock is under constant velocity. The terminal velocity can be obtained from Eq. (1) and (2) by neglecting buoyancy and setting $a = dv/dt = 0$. This gives

$$mg - bv_T^n = 0 \rightarrow v_T = (\frac{mg}{b})^{1/n} \quad \dots \dots \dots \dots (3)$$

Here, measuring the parameters b and n could be replaced measuring its terminal velocity v_T. This helps us to find the eligible trajectory of the shuttlecock.

Resistance force could be modeled in two ways, either proportional to object speed or to the speed squared. Assume the shuttlecock is hit with initial velocity $\vec{v}_i$, then, the horizontal and vertical velocities are expressed by

$$v_{xi} = v_i \cos\theta_i, v_{yi} = v_i \sin\theta_i \quad \dots \dots \dots \dots (4)$$

where angle θ_i is the initial angle.

$n = 1$, we have $\vec{F}_v = F_{vx}\hat{i} + F_{vy}\hat{j}$ $\quad \dots \dots \dots \dots (5)$

$F_{vx} = bv_x$ *and* $F_{vy} = bv_y$ *are the x - component and y - component of air resistance.*

We consider vertical and horizontal directions respectively.
    (a) Vertical component:

$$-mg - bv_y = m\frac{dv_y}{dt} \quad \dots \dots \dots \dots (6)$$

The value of v_y is "+" for upward and "−" for downward. Then we may integrate Eq. (6) to obtain vertical velocity.

$$v_y(t) = (v_t + v_{yi})e^{-\frac{gt}{v_t}} - v_t \quad \dots \dots \dots \dots (7)$$
    and height

$$y(t) = \frac{v_t}{g}(v_t + v_{yi})(1 - e^{-\frac{gt}{v_t}}) - v_t t \quad \dots \dots \dots \dots (8)$$

Where m is the mass of shuttlecock, v_{yi} is the initial vertical velocity and we have taken y = 0 at t = 0, g is the gravitational acceleration, and $v_t = \frac{mg}{b}$ is the terminal velocity.

From Eq. (7), at the top of trajectory, that is, the point of maximum height H, we have $v_y = 0$, this gives the time of flight

$$t = \frac{v_t}{g} \ln \frac{v_t + v_{yi}}{v_t} \quad \dots \dots \dots \dots (9)$$

(b) Horizontal component:

$$-bv_x = m\frac{dv_x}{dt} \quad \dots \dots \dots \dots (10)$$

We may integrate Eq. (10) to obtain horizontal velocity

$$v_x = v_{xi}e^{-\frac{gt}{v_t}} \quad \dots \dots \dots \dots (11)$$

and horizontal distance

$$x = \frac{v_t v_{xi}}{g}(1 - e^{-\frac{gt}{v_t}}) \quad \dots \dots \dots \dots (12)$$

Combine Eq. (8) and (12), then we have the equations of the trajectory as

$$y = x\frac{v_t + v_{yi}}{v_{xi}} - \frac{v_t^2}{g}\ln\frac{v_t v_{xi}}{v_t v_{xi} - gx} \quad \dots \dots \dots \dots (13)$$

$n = 2$, consider the air resistance force of the shuttlecock as $\vec{F}_v = F_{vx}\hat{i} + F_{vy}\hat{j}$, here $F_{vx} = bv^2\cos\theta$ and $F_{vy} = bv^2\sin\theta$ are the x - component and y - component of air resistance force, respectively. Obviously, the factor v in each of the force component expressions is the essential coupling between the x and y equations that prevents any analytic solutions from being found. These equations can only be solved satisfactorily accurately using analytical method. But in order to describe the role of terminal velocity and approach the practical trajectory of a shuttlecock, let us consider motion in the vertical and horizontal directions separately.

A. Vertical directional motion: in this case

$$F_v = F_{vy} = bv_y^2 - mg - bv_y^2 = m\frac{dv_y}{dt} \quad \dots \dots \dots \dots (14)$$

After integration, we have the solution of vertical velocity as

$$v_y = [v_{yi} - v_t'\tan(\frac{gt}{v_t'})]/[1 + \frac{v_{yi}}{v_t'}\tan(\frac{gt}{v_t'})] \quad \dots \dots \dots (15)$$

Where $v_t' = (\frac{mg}{b})^{1/2}$ is the terminal velocity. When the shuttlecock reaches the highest point where we have $v_y = 0$, , the time of flight 't' is expressed as follows:

$$t = \frac{v_t'}{g}\tan^{-1}(\frac{v_{yi}}{v_t'})_{yi} \quad \dots \dots \dots \dots (16)$$
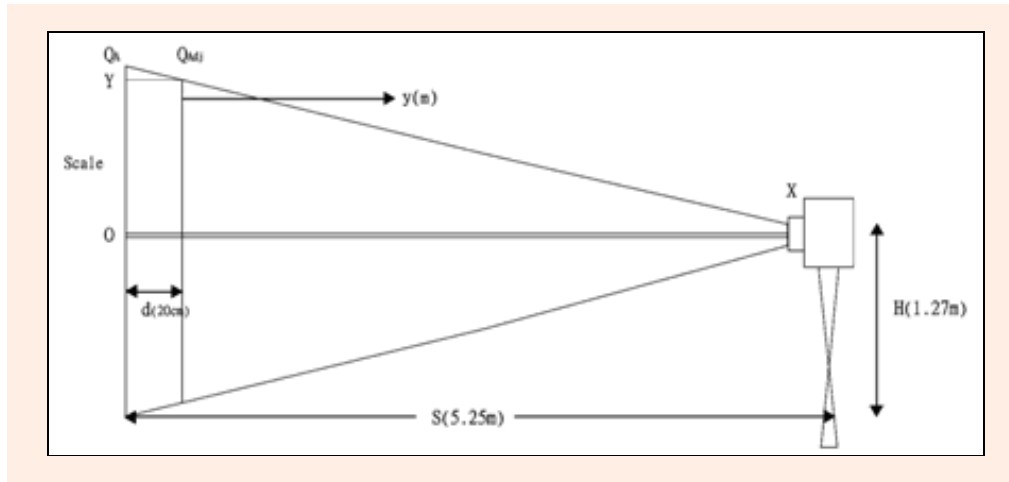
Figure 1. Experimental schematic diagram of y(t) measurement of shuttlecock.

At this moment, the height of apex is

$$y = \frac{v_t'^2}{g} \ln \left| \frac{\sin[\frac{gt}{v_t'} + \tan^{-1}(\frac{v_t'}{v_{yi}})]}{\sin[\tan^{-1}(\frac{v_t'}{v_{yi}})]} \right| \quad \dots\dots\dots\dots\dots (17)$$

B. Horizontal direction: here we have

$$F_v = F_{vx} = bv_x^2 \text{ and } -bv_x^2 = m\frac{dv_x}{dt} \quad \dots\dots\dots (18)$$

and could find the horizontal speed as

$$v_x = \frac{v_{xi} v_t'^2}{v_{xi} gt + v_t'^2} \quad \dots\dots\dots\dots\dots\dots\dots (19)$$

and horizontal distance as

$$x = \frac{v_t'^2}{g} \ln(\frac{v_{xi} gt + v_t'^2}{v_t'^2}) \quad \dots\dots\dots\dots\dots\dots (20)$$

Combine Eq.(17) and (20), we have the equation of the trajectory as

$$y = \frac{v_t'^2}{g} \ln \left| \frac{\sin[\frac{v_t'}{v_{x_i}}(e^{\frac{gx}{v_t'^2}}-1) + \tan^{-1}(\frac{v_t'}{v_{y_i}})]}{\sin[\tan^{-1}(\frac{v_t'}{v_{y_i}})]} \right| \quad \dots\dots (21)$$

Theoretically, the relationship between air resistance and speed was revealed in n and b. At first sight, it seems that to find the trajectory of the shuttlecock, we need to measure n and b in the beginning. However, from the derived results of Eq. (13) or Eq. (21), we have found that the coefficients n and b could be determined by the terminal velocity v_t. In other words, if the terminal velocity was measured, the shuttlecock's trajectory might be found.

Results

Figure 1 is the experimental schematic diagram, where X is the video camera, Q_A is the initial shuttlecock position, Q_{Adj} is the position after parallax correction, and d is the distance between the shuttlecock and scale. We found that the terminal velocity of some shuttlecocks ranged from
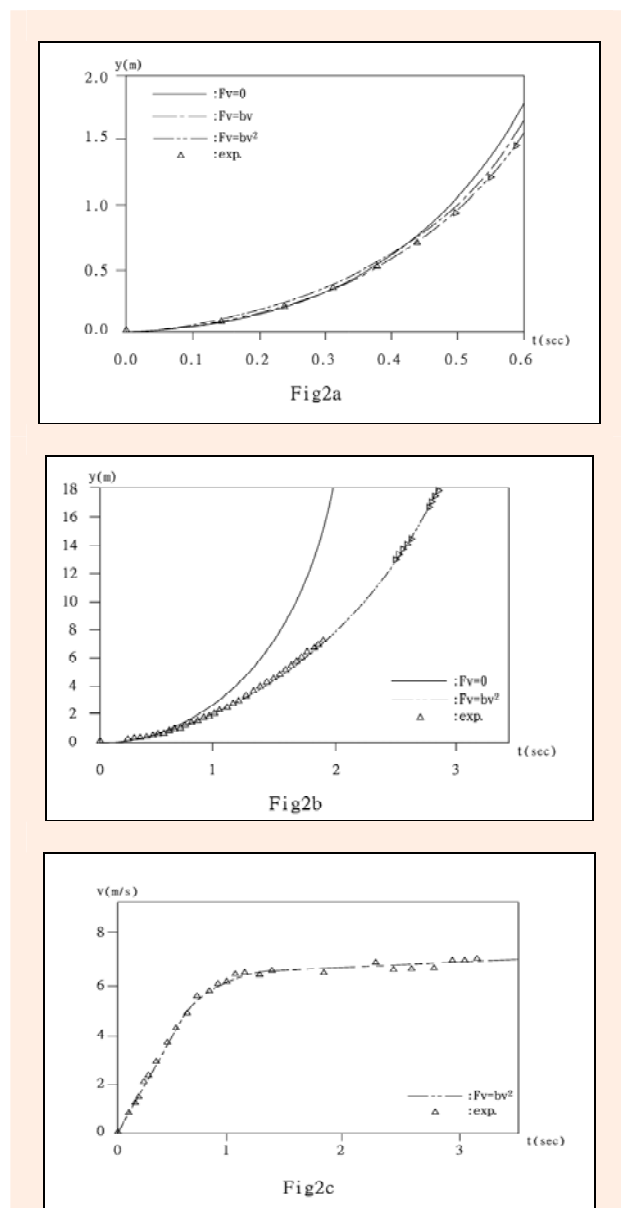

Figure 2. (a) Experimental and calculated y(t)-t diagram, (b) Experimental and calculated y(t)-t diagram, (c) Experimental and calculated quadratic air resistance force v(t)-t diagram.

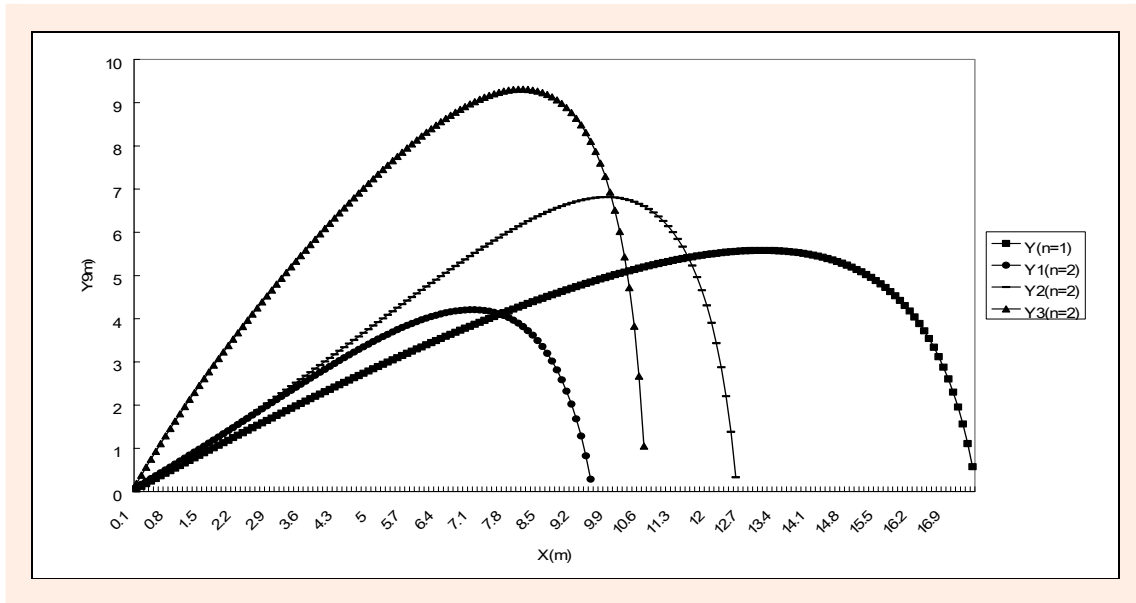**Figure 3.** Theoretical trajectories of shuttlecock.

6.51 to 6.87 m·s⁻¹. One of the experimental shuttlecocks has the mass of 5.19 g, a head diameter of 2.70 cm and neck diameter of 4.20 cm, a tail diameter of 6.50 cm and a head-to-tail length of 9.00 cm. The shuttlecock was released from a height of 18 m, and experimental y(t) was measured. In Figure 2a and 2b, it was found that the best equation for measuring and calculating a shuttlecock trajectory was that of the quadratic air resistance force. From the measured data, we could plot the v(t)-t diagram in Figure 2c and the terminal velocity 6.86 m·s⁻¹ was found. This is very close to the value 6.80 m·s⁻¹ that Peastrel, Lynch and Angelo (1980) had measured. Figure 3 and 4 are the theoretical trajectories from Eq. (13) and (21). Where Y (n = 1) and Y1(n = 2) in Figure 3 both have same initial velocity v_i = 30 m·s⁻¹ $\theta_i = 30^0$ but consider the different condition tha $F_V = bv$ and $F_V = bv^2$ respectively. We found the shuttlecock landed with a horizontal distance of 17.48 m and 9.65 m respectively. Therefore, the shuttlecock in the first case will land outside of the standard court, which has a length of 13.41 m (44 ft). Generally, in practice, the initial speed of a shuttlecock in overhead stroke hit is below 56 m·s⁻¹. In other words, at $\theta_i = 30^0$ the initial speed of most players is often greater than v_i = 30 m·s⁻¹. Very often, the flying shuttlecock lands inside the court. This also helps us to judge intuitionally. What is the relationship between air drag resistance force and the speed of a shuttlecock that have been described above. Apparently, the possibility of the first condition, which states that air drag resistance force is proportional to the speed of a shuttlecock (i.e. $F_V = bv$), is very low. Nevertheless, we must point out, in particular, the relationship between air drag resistance force and the speed of a shuttlecock. The trajectory of a shuttlecock could be described by using terminal
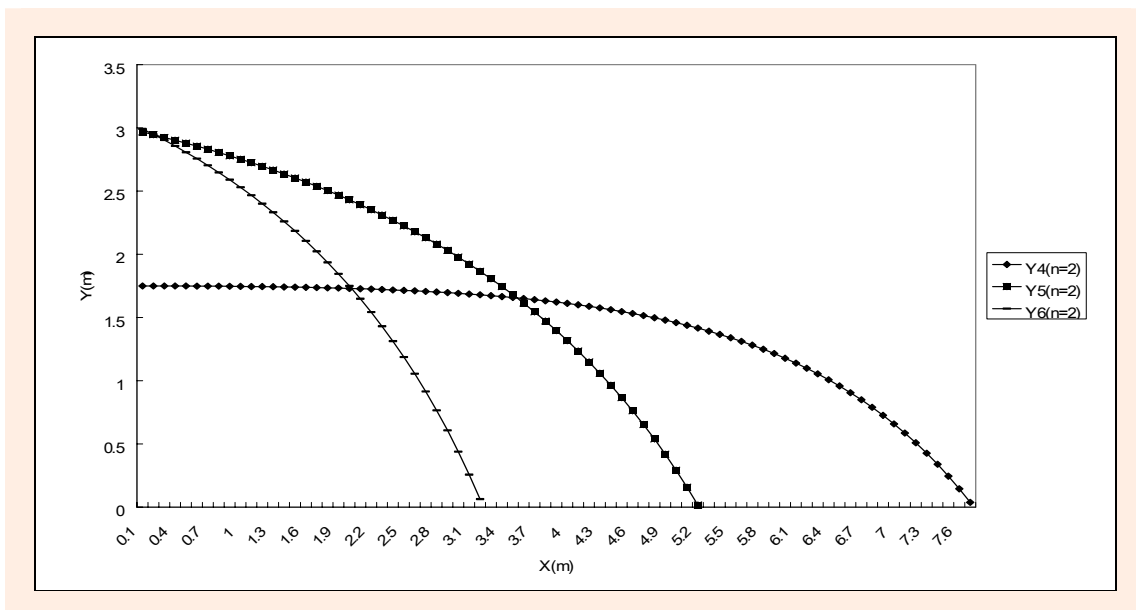

Figure 4. Calculated trajectories of clear, smash and drop of shuttlecock.

velocity. However, we can test again with fast overhead stroke that has an initial velocity of $v_i = 56$ m·s^{-1} $\theta_i = 30^0$ and 60°. Their trajectories Y2 (n = 2) and Y3 (n = 2) are also shown in Figure 3. From Eq. (16) and (17), we also found that the shuttlecock needs 0.88s and 0.94s to reach the top of 6.81m and 9.31m high respectively, and totally 2.33s and 2.63s to alight with a horizontal distance of 12.52m and 10.65m. From Eq. (15) and (19), we could also found that the vertical and horizontal velocities. For example, if the initial angle is adjusted to 60 degree, their vertical and horizontal landing velocities are 6.4 m/s and 1.5 m·s^{-1}. Their ratio is about 4.3 : 1, which interplays the almost vertical landing phenomenon observed. Next in Fig. 4, the trajectories of clear, smash, and drop have their initial velocity of $v_i = 45 m/s, \theta_i = 0^0$, $v_i = 70 m/s, \theta_i = -10^0$ and $v_i = 25 m/s, \theta_i = -20^0$ were shown. Y4 (n=2), Y5 (n=2), and Y6 (n=2) respectively. Their reaction time for received player also could be found if necessary.

Discussion

The special structure of a shuttlecock makes its trajectory perform unsymmetrical motion when playing. From the terminal velocity of a shuttlecock, we could find that the air drag force is proportional to the square of the speed of the shuttlecock. The result is consistent with Peastrel, Lynch and Angelo (1980), who performed an experiment on vertical fall to measure the terminal velocity of a shuttlecock. They found that the best model was the quadratic air resistance force. Tong (2004) performed an identical experiment that also came up with the same result. Furthermore, the angle and strength of a stroke could influence its trajectory. And the equation we found can predict the trajectory of the shuttlecock. Some known values were substituted to check the experimental data. This equation also works adequately on the motion of a shuttlecock under smashing. In this paper, we have considered both the linear and the quadratic air resistance force laws respectively. We found that the trajectory of a shuttlecock could be expressed in terms of its terminal velocity, which means it is unnecessary to find the other parameters, like air drag force. The results revealed that the motion equation of a shuttlecock's flying trajectory could fit experimental data. It also revealed that the angle and force of a stroke could influence trajectory, therefore playing an important part in making strategic plans. This should be helpful for badminton player training. These findings were the same as what Tsai et al. (1995; 1997) had come up with. From the motion equations, it is easy to understand why the order of a shuttlecock speed is: jump-smash, smash, clear, and drop among badminton forehand overhead strokes — because stroke force and angle can affect the trajectory of a shuttlecock significantly.

Moreover, testing a shuttlecock for speed, according to the "Laws of Badminton" by the Badminton World Federation (2006), was specified as law (3.1): to test a shuttlecock, using full underhand stroke which makes contact with the shuttlecock over the back boundary line. The shuttlecock shall be hit at an upward angle and in a direction parallel to the side lines. As law (3.2) suggests, a shuttlecock will land no less than 530 mm and no more than 990 mm short of the other back boundary line as illustrated in Diagram B (not shown). This subjective test is worthy of consideration since the stroke force affects the initial speed, and the initial angle affects the landing distance. For example, if the initial speed is 40 m·s^{-1}, with the release angle of 30 and 60 degrees, their landing distances are 12.50 m and 10.82 m, respectively. Therefore, an objective and revised test is necessary. As an illustration, if we let a shuttlecock fall from a height of 5 m, it will land on the ground after ~1.20 s.

Conclusion

From the terminal velocity of a shuttlecock, the conclusion reveals that the equation of this study could predict the trajectory of a shuttlecock, and it shows that air drag force is proportional to the square of a shuttlecock velocity. Additionally, the angle and strength of a stroke could also influence trajectory. Finally, this study presented a suggestion that we could use a scientific approach to test a shuttlecock's velocity objectively and to replace the traditional and subjective method made by the Badminton World Federation. Finally, research findings can be applied to improving the professional knowledge of badminton player training.

References

Adrian, M.J. and Enberg, M.L. (1971*) Sequential timing of three overhand patterns. Kinesiology review.* VA: AAHPERD. 1-9.

Chang, S.S. (2002) *Kinematical analysis via three - dimensional cinematography for two types of forehand smash stroke in senior high school badminton players.* Master thesis, National Taiwan Normal University, Taiwan. (In Chinese: English abstract).

Gowizke, B.A. and Waddell, K.B. (1979) *Technique of badminton stroke production: science in badminton.* In: Racquet Sports. Del Mar, CA Academic.

Hsiao, P.R. (2005) *The Route analyzing of Badminton Rearcourt Strokes by International women's single players.* Master thesis, National Taiwan Normal University, Taiwan. (In Chinese: English abstract).

Badminton World Federation. (2006) The simplified laws of Badminton. February. Available from URL: http://www.internationalbadminton.org.

Peastrel, M., Lynch, R. and Angelo, A. Jr. (1980) Terminal velocity of a shuttlecock in vertical fall, *American Journal of Physics* **48 (7),** 511-513

Poole, J. (1970) *A cinematographic analysis of the upper extremity movement of world class players executing the basic badminton strokes.* Doctoral thesis, Louisiana State University, Louisiana.

Tang, H.P. (1995) Three-dimensional cinematographical analysis of the badminton forehand smash: movement of the forearm and hand. *Science and Racket Sports.* Cambridge: E & FN SPON.

The Engineering Tool Box. (2005) Fluid mechanics, Reynolds number. *An introduction and definition of dimensionless Reynolds Number*, June. Available from URL: http://www.engineeringtoolbox.com/Reynolds-number-d_237.html.

Thornton, S.T. and Marion, J.B.(2003) *Classical dynamics of particles and systems.* Fourth edition. Saunders College Publishing, Harcourt Brace & Company

Tong, G.F. (2004) *Mechanics analysis of a shuttlecock terminal velocity.* Master thesis, National Kaohsiung Normal University, Taiwan. (In Chinese: English abstract).

Tsai, C.L., Huang, C.F. and Jih, S.C. (1997) Biomechanical analysis of four different badminton forehand overhead stokes. *Physical Education Journal* **22,** 189-200.

Tsai, C.L., Huang, C.F. and Jih, S.C. (1995) biomechanical analysis of four different badminton forehand overhead stokes for world class badminton players. *Physical Education and Sport Journal of Taiwan Normal University* **1**, 21-30. (In Chinese: English abstract).

Key points

- The motion equation of a shuttlecock's flying trajectory could be constructed by determining the terminal velocity in aerodynamics.
- Air drag force is proportional to the square of a shuttlecock velocity. Furthermore, the angle and strength of a stroke could influence trajectory.

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